

## *Progress in Lattice QCD at Finite Temperature*

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## Progress in Lattice QCD at finite temperature

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**Abstract.** I review recent developments in lattice QCD at finite temperature, including the determination of the transition temperature  $T_c$ , equation of state and different static screening lengths. The lattice data suggest that at temperatures above  $1.5T_c$  the quark gluon plasma can be considered as gas consisting of quarks and gluons.

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### 1. Introduction

One of the most challenging question in particle and nuclear physics is the one concerning the properties of strongly interacting matter at extremely high temperatures and densities. We expect that at sufficiently high temperatures and densities the strongly interacting matter undergoes a transition to a new state, where quarks and gluons are no longer confined in hadrons, and which is therefore often referred to as a deconfined phase or Quark Gluon Plasma (QGP). We would like to know at which temperature the transition takes place and what is the nature of the transition as well the properties of the deconfined phase, equation of state, static screening lengths, transport properties etc. Lattice QCD can provide first principle calculation of the transition temperature, equation of state and static screening lengths (see Ref. [1,2]) for recent reviews. Calculation of transport coefficients remains an open challenge for lattice QCD (see discussion in Refs. [3,4]).

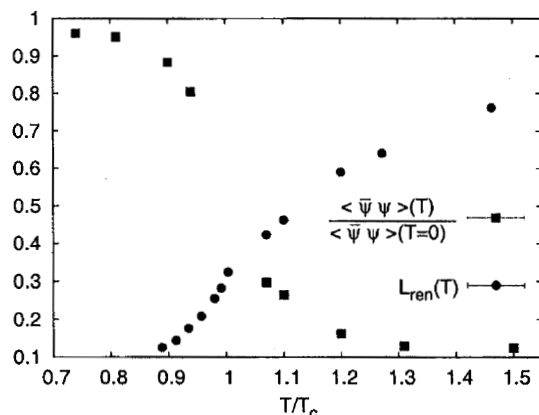
One of the most interesting question for the lattice is the question about the nature of the finite temperature transition and the value of the temperature  $T_c$  where it takes place. For very heavy quarks we have a 1st order deconfining transition. In the case of QCD with three degenerate flavors of quarks we expect a 1st order chiral transition for sufficiently small quark masses. In other cases there is no true phase transition but just a rapid crossover. Lattice simulations of 3 flavor

QCD with improved staggered quarks (p4) using  $N_\tau = 4$  lattices indicate that the transition is first order only for very small quark masses, corresponding to pseudo-scalar meson masses of about 60 MeV [10]. A recent study of the transition using effective models of QCD resulted in a similar estimate for the boundary in the quark mass plane, where the transition is 1st order [8]. This makes it unlikely that for the interesting case of one heavier strange quark and two light  $u, d$  quarks, corresponding to 140 MeV pion, the transition is 1st order. However, calculations with unimproved staggered quarks suggest that the transition is 1st order for pseudo-scalar meson mass of about 300 MeV [11]. Thus the effect of the improvement is significant and we may expect that the improvement of flavor symmetry, which is broken in the staggered formulation, is very important. But even when using improved staggered fermions it is necessary to do the calculations at several lattice spacings in order to establish the continuum limit. Recently, extensive calculations have been done to clarify the nature of the transition in the 2+1 flavor QCD for physical quark masses using  $N_t = 4, 6, 8$  and 10 lattices. These calculations were done using the so-called *stout* improved staggered fermion formulations which is even superior to other more commonly used improved staggered actions (*p4*, *asqtad*) in terms of improvement of flavor symmetry. The result of this study was that the transition is not a true phase transition but only a rapid crossover [12]. Even-though there is no true phase transition in QCD thermodynamic observables change rapidly in a small temperature interval and the value of the transition temperature plays an important role. The flavor and quark mass dependence of many thermodynamic quantities is largely determined by the flavor and quark mass dependence of  $T_c$ . For example, the pressure normalized by its ideal gas value for pure gauge theory, 2 flavor, 2+1 flavor and 3 flavor QCD shows almost universal behavior as function of  $T/T_c$  [9].

The chiral condensate  $\langle \bar{\psi}\psi \rangle$  and the expectation value of the Polyakov loop  $\langle L \rangle$  are order parameters in the limit of vanishing and infinite quark masses respectively. However, also for finite values of the quark masses they show a rapid change in vicinity of the transition temperature. In Figure 1 I show the chiral condensate and the Polyakov loop as function of the temperature calculated for the *p4* action and light quark mass  $m_l = 0.1m_s$ , with  $m_s$  being the physical strange quark mass. Note that in Figure we show the renormalized Polyakov loop defined as in Ref. [13]. The details of the calculations calculated can be found in Ref. [30]. We see that the chiral condensate and the renormalized Polyakov loop show rapid change at  $T_c$  suggesting that the chiral and the deconfinement transitions happen at the same temperature. To determine the value of the transition temperature and to study the interplay between the chiral and the deconfinement transition one usually calculates the disconnected part of the chiral susceptibility and the Polyakov loop susceptibility defined as

$$\frac{\chi_{\bar{\psi}\psi}}{T^2} = N_\sigma^3(\langle (\bar{\psi}\psi)^2 \rangle - \langle \bar{\psi}\psi \rangle^2), \quad \frac{\chi_L}{T^2} = N_\sigma^3(\langle L^2 \rangle - \langle L \rangle^2) \quad (1)$$

as function of the of the bare gauge coupling  $\beta = 6/g^2$ .  $N_\sigma$  is the spatial size

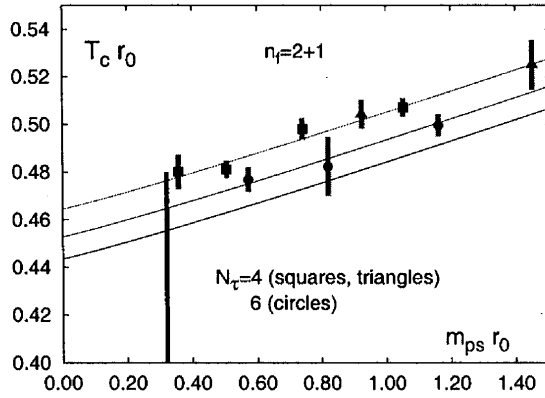


**Fig. 1.** The renormalized Polyakov loop  $L_{ren}(T)$  [30] and the chiral condensate normalized to the zero temperature chiral condensate [15] as function of the temperature calculated on  $16^3 \times 4$  lattices.

of the lattice. The susceptibilities have a peak at some pseudo-critical coupling  $\beta_c$ . The chiral and the Polyakov loop susceptibility have been studied using lattice with temporal extent  $N_\tau = 4$  and  $N_\tau = 6$  and several values of the light quark masses  $m_l = 0.05m_s, 0.1m_s, 0.2m_s$  and  $0.4m_s$  [7]. Note that the smallest value of  $m_l$  correspond to pion masses of about 140MeV. We find that within accuracy of the calculations pseudo-critical couplings  $\beta_c$  determined from the disconnected part of the chiral susceptibility and the Polyakov loop susceptibility coincide. This again shows that the chiral and the deconfinement transition happen at the same temperature. To determine the transition temperature we have to calculate the lattice spacing in terms of some physical quantity. In the past the string tension has been used to set the lattice spacing. A more accurate determination of the lattice spacing is provided by the so-called Sommer scale  $r_0$  defined from the static quark anti-quark potential as

$$r^2 \frac{dV(r)}{dr} \Big|_{r=r_0} = 1.65. \quad (2)$$

Analysis of the quarkonium spectroscopy on the lattice lead to the value  $r_0 = 0.469(7)\text{fm}$  [16]. In figure 2 I show the transition temperature in units of  $r_0$  for different quark masses [7] and two different lattice spacings, corresponding to  $N_\tau = 4$  and  $N_\tau = 6$  lattices. Note that the value of  $T_c$  calculated at two different lattice spacings are clearly different. The thin error-bars in Figure 2 represent the error in the determination of the lattice spacing  $a$ , i.e. the error in  $r_0/a$ . There is also an error in the determination of the gauge coupling constant  $\beta_c = 6/g^2$ . The combined error is shown in Fig. 1b as a thick error-bar. For  $N_t = 4$  calculations the error is dominated by the error in lattice spacing, while for  $N_t = 6$  it is dominated by the error in  $\beta_c$ . With the data on  $r_0 T_c$  a chiral and continuum extrapolation



**Fig. 2.** The transition temperature in units of the  $r_0 T_c$  from Ref. [7] as function of the pion mass.

has been attempted using the most simple Ansatz  $r_0 T_c(m_\pi, N_t) = r_0 T_{c|cont}^{chiral} + A(r_0 m_\pi)^d + B/N_t^2$ . From this extrapolation one gets the continuum value  $T_c r_0 = 0.457(7)[+8][-2]$  for the physical pion mass  $m_\pi r_0 = 0.321$  [7]. The central value was obtained using  $d = 1.08$  expected from  $O(4)$  scaling. To test the sensitivity to the chiral extrapolations  $d = 2$  and  $1$  have also been used. The resulting uncertainty is shown as second and third error in square brackets. Using the best known value of  $r_0 = 0.469(7)\text{fm}$  we obtain  $T_c = 192(7)(4)\text{ MeV}$  which is higher than the most of the previous values. It is also significantly higher than the chemical freezeout temperature at RHIC [17]. Note that the large value of the transition temperature is mostly due to the large value of the string tension which is related to the Sommer scale as  $r_0 \sqrt{\sigma} = 1.114(4)$  [18]. Using the above value of  $r_0$  we get  $\sqrt{\sigma} = 468\text{ MeV}$  which is more than 10% larger than the value  $\sqrt{\sigma} = 420\text{ MeV}$  which was used in Ref. [19] and led to  $T_c = 173(8)\text{ MeV}$ .

Lattice calculations of equation of state were started some twenty years ago. In the case of QCD without dynamical quarks the problem has been solved, i.e. the equation of state has been calculated in the continuum limit [20]. At temperatures of about  $4T_c$  the deviation from the ideal gas value is only about 15% suggesting that quark gluon plasma at this temperature is weakly interacting. Perturbative expansion of the pressure, however, showed very poor convergence at this temperature [21]. Only through the use of new resummed perturbative techniques it was possible to get agreement with the lattice data [22–24]. To get a reliable calculation of the pressure and the energy density improved action have to be used [25, 26]. Very recently calculations with the so-called *asqtad* and *p4* action have been done on lattices with temporal effects  $N_\tau = 4$  and  $6$  [27, 28]. In Figure 3 the interaction measure  $\epsilon - 3p$  is shown as function of the temperature for the *p4* action. Calculations performed for  $N_\tau = 4$  and  $N_\tau = 6$  give similar results. This means that cutoff effects are under control. Furthermore, there is a good agreement between *p4* and *asqtad*

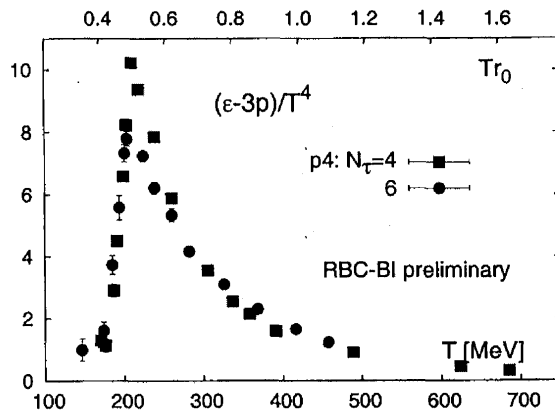


Fig. 3. The interaction measure calculated for the p4 action [28].

calculations. We see that close to  $T_c$  the interaction measure is very large, which means that quark gluon plasma at this temperature is very far from the conformal limit. At high temperature the value of the interaction measure is consistent with the perturbative estimate.

## 2. Spatial correlation functions

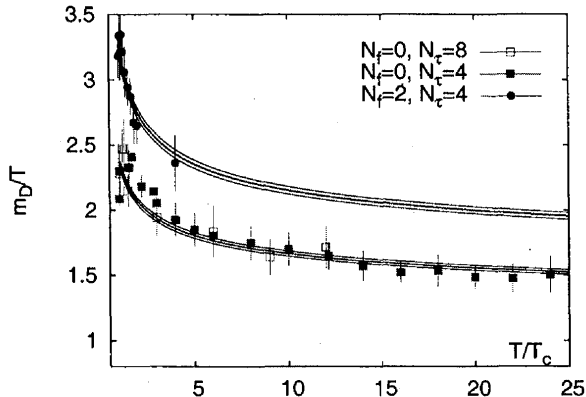
To get further insight into properties of the quark gluon plasma one can study different spatial correlation functions. One of the most prominent feature of the quark gluon plasma is the presence of chromoelectric (Debye) screening. The easiest way to study chromoelectric screening is to calculate the singlet free energy of static quark anti-quark pair (for recent review on this see Ref. [29]), which is expressed in term of correlation function of temporal Wilson lines

$$\exp(-F_1(r, T)/T) = \text{Tr}\langle W(r)W^\dagger(0) \rangle. \quad (3)$$

$L = \text{Tr}W$  is the Polyakov loop. In absence of dynamical quarks the free energy grows linearly with the separation between the heavy quark and anti-quark in the confined phase. In presence of dynamical quarks the free energy is saturated at some finite value at distances of about 1 fm due to string breaking [29, 30]. Above the deconfinement temperature the singlet free energy is exponentially screened, at sufficiently large distances [31], i.e.

$$F_1(r, T) = F_\infty(T) - \frac{4}{3} \frac{g^2(T)}{4\pi r} \exp(-m_D(T)r). \quad (4)$$

The inverse screening length or equivalently the Debye screening mass  $m_D$  is proportional to the temperature. In leading order of perturbation theory it is  $m_D = \sqrt{1 + N_f/3} g(T)T$ . Beyond leading order it is sensitive to the non-perturbative dy-

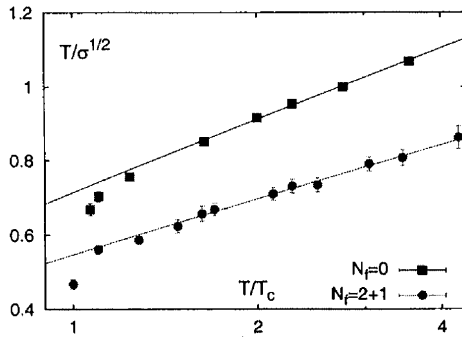


**Fig. 4.** The Debye mass calculated in quenched QCD [31] and 2 flavor QCD [32]. The lines show the leading order fit together with its uncertainty.

namics of the static chromomagnetic fields. The Debye screening mass has been calculated in pure gauge theory ( $N_f = 0$ ) [31] and in 2 flavor QCD ( $N_f = 2$ ) [32] and is shown in Figure 4 at different temperatures. The temperature dependence of the lattice data have been fitted with the simple Ansatz motivated by the leading order result :  $m_D(T) = A\sqrt{1 + N_f/3g(T)}T$ . Here  $g(T)$  is the two loop running coupling constant. This simple form can fit the data very well and we get  $A \simeq 1.4$  both for  $N_f = 0$  and  $N_f = 2$ . Thus the temperature dependence as well as the flavor dependence of the Debye mass is given by perturbation theory. We also see that non-perturbative effects due to static magnetic fields significantly effect the electric screening, resulting in about 40% corrections. However, the non-perturbative correction is the same in full QCD and pure gauge theory. Let us note that in SU(2) gluodynamics the corrections to the Debye mass are even larger, the Debye mass is 1.6 times larger than the leading order result [33–35]. This situation can be understood in terms of dimensionally reduced effective theory, where the effect of hard modes with momentum  $p \sim \pi T$  is integrated out and which contain only static electric and magnetic fields [36]. The validity of dimensional reduction has been tested in a wide temperature range [34, 35].

At zero temperature the static quark anti-quark potential is determined from the Wilson loops:  $V(r) = -1/t \ln W(r, t)$ ,  $t \rightarrow \infty$ . At large separation the Wilson loop obeys the area law  $W(r, t) \sim \exp(-\sigma r t)$  which means that the potential grows linearly with distance  $r$ . At finite temperature we can consider the spatial Wilson loops. They obey area law at any temperature  $W_s(x, z) \sim \exp(-\sigma_s(T)xz)$  [37, 38]. Below the transition temperature the spatial string tension is very close to the usual zero temperature string tension. Well above the deconfinement transition temperature the spatial string tension is expected to be  $\sqrt{\sigma_s(T)} = c_M g^2(T)T$  [38]. This is because in dimensionally reduced theory it is given by  $c_M g_3^2$  and at leading order the 3-dimensional gauge coupling is  $g_3^2 = g^2(T)T$ . The spatial string tension





**Fig. 5.** The spatial string tension calculated in quenched QCD [39] and 2 flavor QCD [40]. The lines show the leading order fits.

has been calculated on the lattice in quenched QCD ( $N_f = 0$ ) [39] and 2+1 flavor QCD [40] and the results are shown in Figure 5. The lattice data can be fitted very well with the simple form :  $\sqrt{\sigma_s(T)} = c_M g^2(T) T$ . Here again  $g(T)$  is the 2-loop running coupling. For the fit we get the value of  $c_M$  which agrees well with the result of dimensional reduction [40]. The 3-dimensional gauge coupling  $g_3^2$  has been calculated more systematically in perturbation theory and also led to a very good agreement with the lattice data [41].

### 3. Conclusions

In recent years significant progress has been made in calculating bulk thermodynamic observables on the lattice as well as spatial correlation functions. This calculations suggest that at temperatures  $T > 1.5T_c$  thermodynamics can be described reasonably well using weak coupling approaches: resummed perturbation theory and dimensional reduction. The temperature and flavor dependence of static screening length is well described by perturbation theory. However, the value of the screening lengths is to large extent non-perturbative and influenced or determined by the dynamics of static magnetic fields. Furthermore, there is no evidence for the large value of the gauge coupling constant at scale  $T$ . Clearly more precise lattice data and further perturbative calculations are needed to establish the nature of quark gluon plasma in the temperature,  $T > 1.5T_c$ .

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